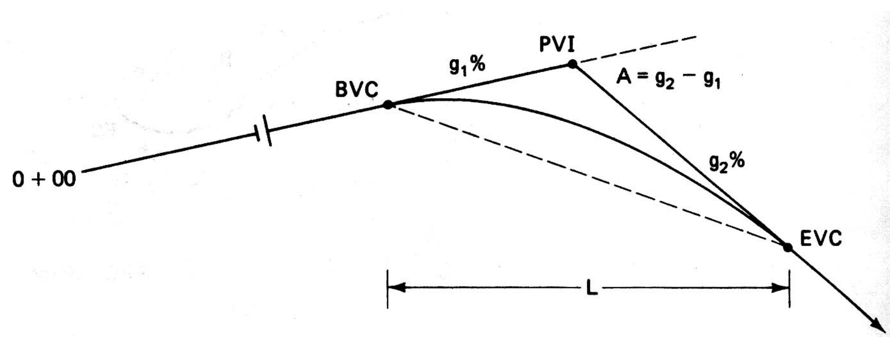
VERTICAL CURVES

GENERAL

Vertical curves are used in highway and street vertical alignment to provide a gradual change between two adjacent grade lines. Some highway and municipal agencies introduce vertical curves at every change in grade line slope, whereas other agencies introduce vertical curves into the alignment only when the net change in slope direction exceeds a specific value (e.g., 1.5 percent or 2 percent).

In figure below, vertical curve terminology is introduced: g_1 is the slope (percent) of the lower station grade line. g_2 is the slope of the higher station grade line**. BVC** is the beginning of the vertical curve. EVC is the end of the vertical curve. PVI is the point of intersection of the two adjacent grade lines. The length of vertical curve *(L)* is the projection of the curve onto a horizontal surface and as such corresponds to plan distance.



Vertical curve terminology

The algebraic change in slope direction is A, where

$$A = g_2 - g_1$$

Example.

if g_1 = + 1. 5 percent and g_2 = -3.2 percent,

A would be equal to (- 3.2 - 1.5) = -4.7.

Note a vertical curve is required if A -is greater than 2.0 or less than -2.0

The geometric curve used in vertical alignment design is the vertical axis parabola. The parabola has the desirable characteristics of

1. A constant rate of change of slope, which contributes to smooth alignment transition
2. Ease of computation of vertical offsets, which permits easily computed curve elevations.

The general equation of the ***parabola*** is

$$y = ax^2 + bx + c$$

The slope of this curve at any point is given by the first derivative,

$$dy/dx = 2ax + b$$

The rate of change of slope is given by the second derivative,

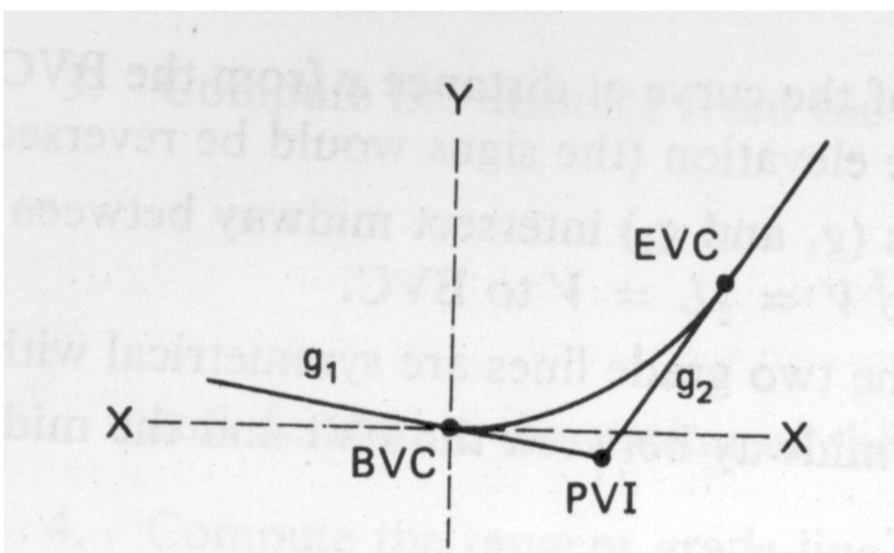
$$d^2y/dx^2 = 2a$$

2a is a constant.

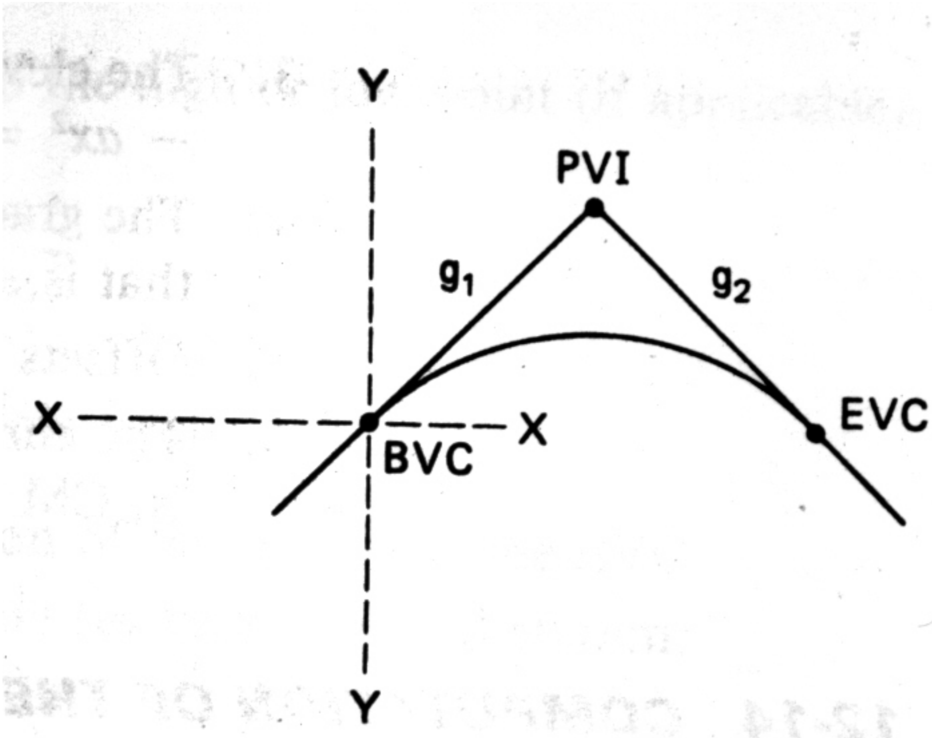
The rate of change of slope (*2a*) can also be written as A/L.

Change of Axis

Placed the origin of the axis at BVC



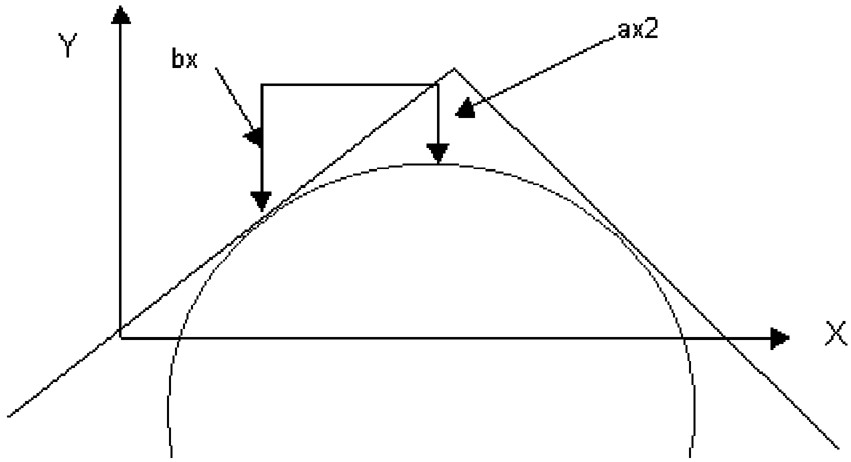
Sag Curve



Crest Curve

The general equation becomes:

$$y = ax^2 + bx$$



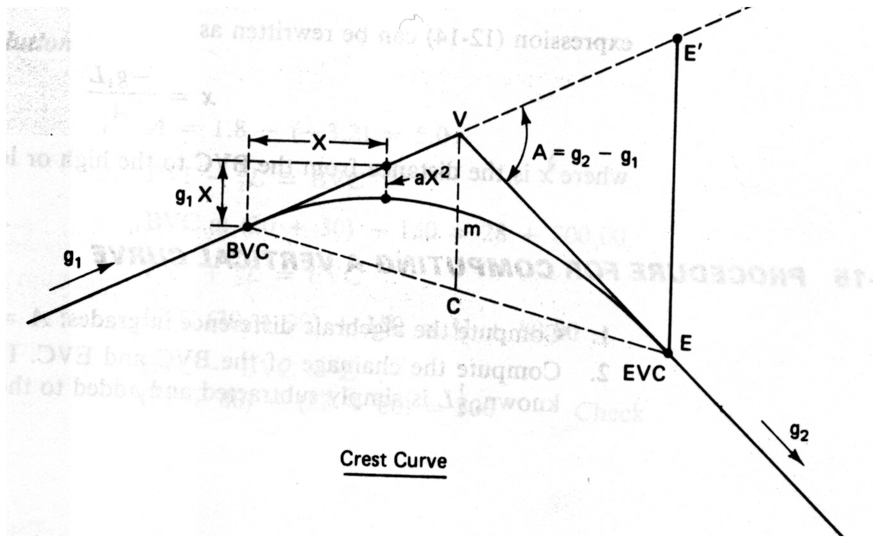
The slope at the origin is g_1, the expression for slope of the curve becomes

$$dy/dx = slope = 2ax + g_1$$

The general equation is finally be written:

$$y = ax^2 + g_1x$$

Properties of a Vertical Curve



Crest Curve

1. The difference in elevation between the BVC and a point on the g_1 grade line at a distance X units (feet or meters) is g_1X (g_1 is expressed as a decimal).
2. The tangent offset between the grade line and the curve is given by ax^2, where x is the horizontal distance from the BVC; (that is, tangent offsets are proportional to the squares of the horizontal distances).
3. The elevation of the curve at distance X from the BVC is given (on a crest curve) by:

$$BVC + g_1x - ax^2$$

(the signs would be reversed in a sag curve).

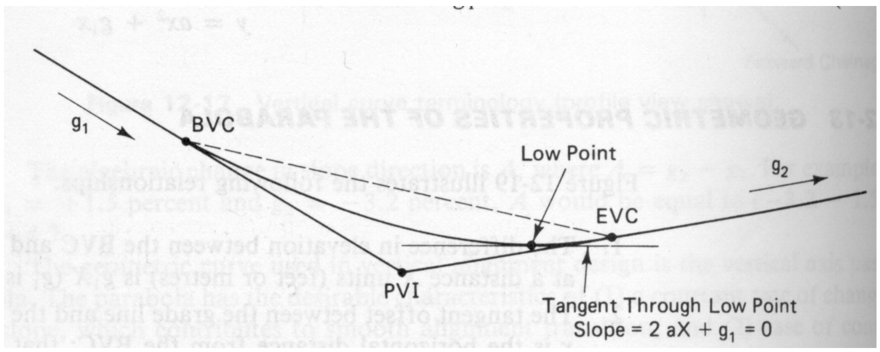
4. The grade lines (g_1 and g_2) intersect midway between the BVC and the EVC. That is, BVC to V = 1/2L = V to EVC.
5. Offsets from the two grade lines are symmetrical with respect to the PVI.
6. The curve lies midway between the PVI and the midpoint of the chord; that is, Cm = mV.

Computation of Low or High Point on Curve

The locations of curve high and low points are important for drainage and bridge considerations. For example, on curbed streets catch basins must be installed precisely at the drainage low point.

It was noted earlier that the slope was given by the expression

$$\text{Slope} = 2ax + g_1$$



The figure above shows a sag vertical curve with a tangent drawn through the low point; it is obvious that the tangent line is horizontal with a slope of zero; that is,

$$2ax + g_1 = 0$$

Since $2a = A/L$

$$x = -g_1 L/A$$

where x is the distance from the BVC to the high or low point.

Procedure for Computing a Vertical Curve

1. Compute the algebraic difference in grades: *A*
2. Compute the chainage of the BVC and EVC. If the chainage of the PVI is known, *1/2L* is simply subtracted and added to the PVI chainage.
3. Compute the distance from the BVC to the high or low point (if applicable):

$$x = -g_1 L/A$$

and determine the station of the high/low point.

4. Compute the tangent grade line elevation of the BVC and the EVC.
5. Compute the tangent grade line elevation for each required station.

6. Compute the midpoint of chord elevation

$$\{\text{elevation of BVC} + \text{elevation of EVC}\}/2$$

7. Compute the tangent offset (d) at the PVI (i.e., distance Vm):

$$d = \{\text{elevation of PVI} - \text{elevation of midpoint of chord}\}/2$$

8. Compute the tangent offset for each individual station.

$$\text{Tangent offset} = \{x/(L/2)\}^2 d$$

where x is the distance from the BVC or EVC (whichever is closer) to the required station.

9. Compute the elevation on the curve at each required station by combining the tangent offsets with the appropriate tangent grade line elevations. Add for sag curves and subtract for crest curves.

Example

Given that $L = 300$ ft, $g_1 = -3.2\%$, $g_2 = +1.\ 8\%$, PVI at 30 + 30, and elevation = 465.92. Determine the location of the low point and elevations on the curve at even stations, as well as at the low point.

Solution

$$A = g_2 - g_1 = 1.8 - (-3.2) = 5.0$$

$$\text{PVI} - 1/2\text{L} = \text{BVC}$$

$$30+30 - 150 = 28+80.00$$

$$\text{PVI} + 1/2\text{L} = \text{EVC}$$

$$30+30 + 150 = 31+80.00$$

$$\text{EVC} - \text{BVC} = \text{L}$$

$$31+80 - 28+86 = 300\text{….Check}$$

$$\text{Elevation of PVI} = 465.92$$

$$150 \text{ ft at } 3.20\% = 4.80\text{ft}$$

Elev. BVC = 465.92 + 4.80 = 470.72

Elevation PVI = 465.92

150 ft at 1.8% = 2.70

Elevation EVC = 465.92 + 2.70 = 468.62

Location of low point

$x = -g_1 L/A$

{3.2 x 300}/5 = 192.00 ft

Tangent grade line computations

elevation at 29 + 00 = 470.72 - (0.032 x 20)

= 470.72 - 0.64 = 470.08

Mid-chord elevation:

{470.72 (BVC) + 468.62 (EVC)}/2 = 469.67

Tangent offset at PVI (d):

d = {elevation of PVI - elevation of mid-chord}/2

{469.67 - 465.92}/2 = 3.75/2 = 1.875ft

Point	Station	Tangent Elevation	Tangent Offset $\{x/(L/2)\}^2 d$	Curve Elevation
BVC	28 + 80	470.72.	$(0/150)^2$ X 1.875 = 0	470.72
	29 + 00	470.08	$(20/150)^2$ X 1.875 = 0	470.11
	30 + 00	466.88	$(120/150)^2$ X 1.875 = 0	468.08
PVI	30 + 30	465.92	$(150/150)^2$ X 1.875 = 0	467.80
Low Point	30 + 72	466.68	$(108/150)^2$ X 1.875 = 0	467.65
	31 + 00	467.18	$(80/150)^2$ X 1.875 = 0	467.71
EVC	31 + 80	468.62	$(0/150)^2$ X 1.875 = 0	468.62

Parabolic Curve Elevations by Tangent Offsets